## A Constraint-free Approach to Optimal Reinsurance

Hans U. Gerber<sup>1,3</sup>, Elias S. W. Shiu<sup>2</sup>, Hailiang Yang<sup>1</sup>

<sup>1</sup>The University of Hong Kong, Hong Kong, China <sup>2</sup>The University of Iowa, Iowa City, U.S.A. <sup>3</sup>The University of Lausanne, Lausanne, Switzerland

## Abstract

Reinsurance is available for a reinsurance premium that is determined according to a convex premium principle H. The first insurer selects the reinsurance coverage that maximizes his expected utility. No conditions are imposed on the reinsurer's payment. The optimality condition involves the gradient of H. For several combinations of H and the first insurer's utility function, closed form formulas for the optimal reinsurance are given. If H is a zero utility principle (for example, an exponential principle or an expectile principle), it can be shown, with results from Pareto optimal risk exchanges and the Theorem of Borch, that the optimal reinsurer's payment satisfies the conditions that usually have to be imposed.